Lecture 4

Material and Homework

Read:

Chapter 6 Yariv and Yeh

Appendices 1 and 2 in Coldren (on reserve)

Homework #2

1. Derive the expression for mode spacing in a Fabry-Perot laser.

2. Problem 6.1 Yariv

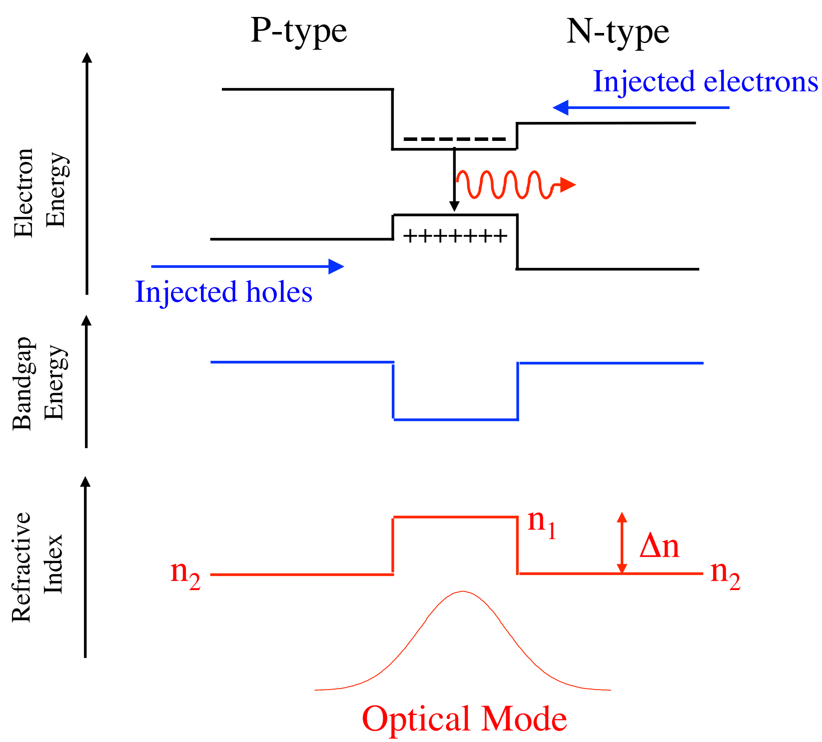
3. Problem 6.2 Yariv

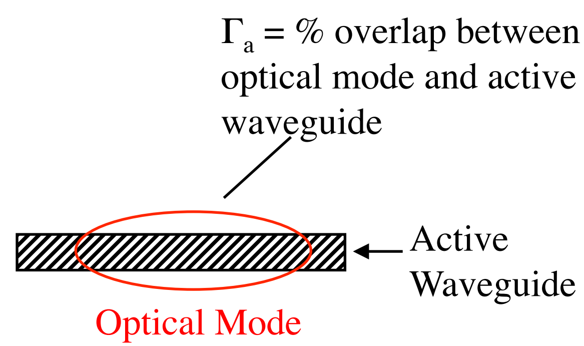
4. Consider a 1.55 μm InGaAsP /InP bulk laser 400 μm in length with confinement factor Γ=.05, internal quantum efficiency of 80% and internal loss of 10 cm^{-1}. cleaved facets, 0.2 mm thick active region, 0.1 ps lifetime for the linewidth calculation.

- Plot the gain versus wavelength
- Plot the peak gain versus carrier density
- What is the mirror loss?
- What is the threshold modal gain?
- What is the differential quantum efficiency?
- What is the axial mode spacing?

Optical and Carrier Confinement

- A heterostructure is a p-n junction between materials with dissimilar bandgaps
- Used to confine: Carriers (efficiency) and Photons (waveguide)





Laser Rate Equations

⇨ Define the laser output power P(t), the current I(t), the active gain volume V, and the carrier and photon densities N(t) and S(t) respectively.

⇨ The dynamics of carrier and photon density in the semiconductor laser cavity is governed by couple rate equations

Carrier density

Optical gain

Nonlinear gain

$$\frac{dN}{dt} = \frac{I}{qV} - \frac{N}{\tau_n} - G(N)\cdot(1-\varepsilon\cdot S)\cdot S$$

Photon density

Spontaneous emission factor

$$\frac{dS}{dt} = \Gamma_a \cdot G(N)\cdot(1-\varepsilon\cdot S)\cdot S - \frac{S}{\tau_p} + \frac{\Gamma_a \beta_{sp} N}{\tau_n}$$

Optical confinement factor

Carrier lifetime

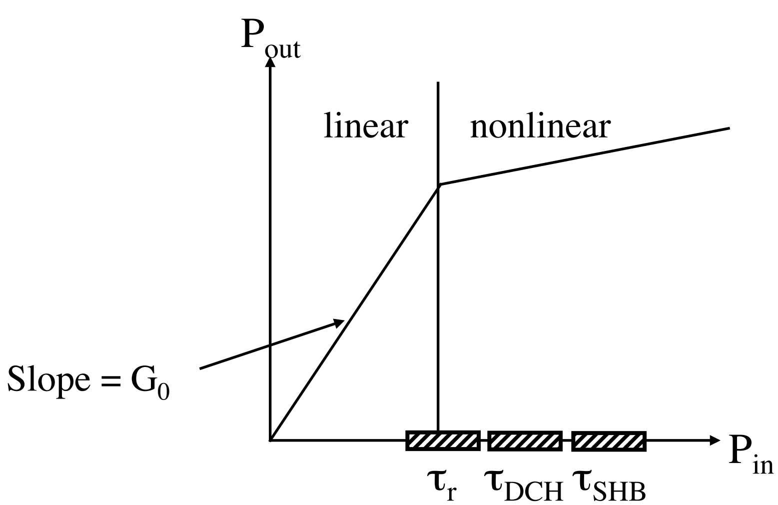
Photon lifetime

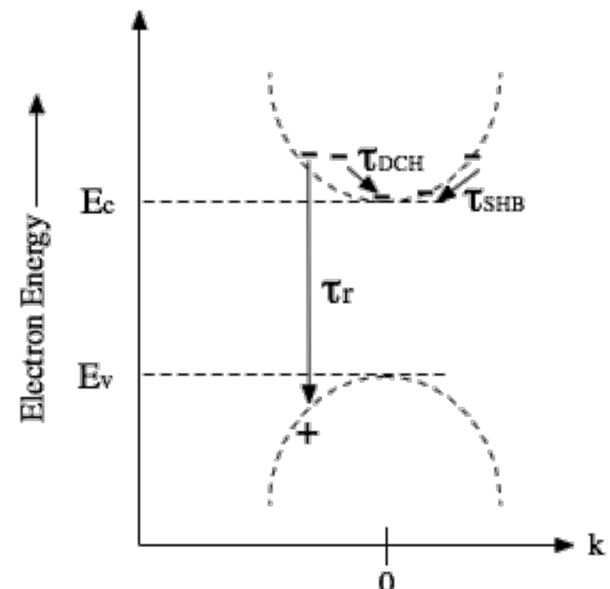
Nonlinear Gain (Gain Compression)

- Gain saturation (recovery) is governed by the rate that carriers can be replenished.
  - Band to band is governed by (τ_r)
  - Carriers cool from levels within band to band-edge by giving up energy to phonons (τ_{DCH})
  - Carriers scatter off one another, changing momentum and energy along the band-edge (τ_{SHB})

$$\varepsilon = \frac{1}{\tau_r} + \frac{1}{\tau_{DCH}} + \frac{1}{\tau_{SHB}}$$

$$\tau_r \approx 1ns, \tau_{DCH} \approx 650\,fs, \tau_{SHB} \approx 50\,fs$$





Steady State Solutions to Rate Equations

⇨ Setting the rate equations equal to zero

$$\frac{dN}{dt} = \frac{dS}{dt} = 0$$

⇨ and the cavity photon density S=0, we get the below (at) threshold condition

$$\frac{I_{th}}{qV} = \frac{N_{th}}{\tau_n}$$

$$I_{th} = \frac{qVN_{th}}{\tau_n} = qVR(N_{th})$$

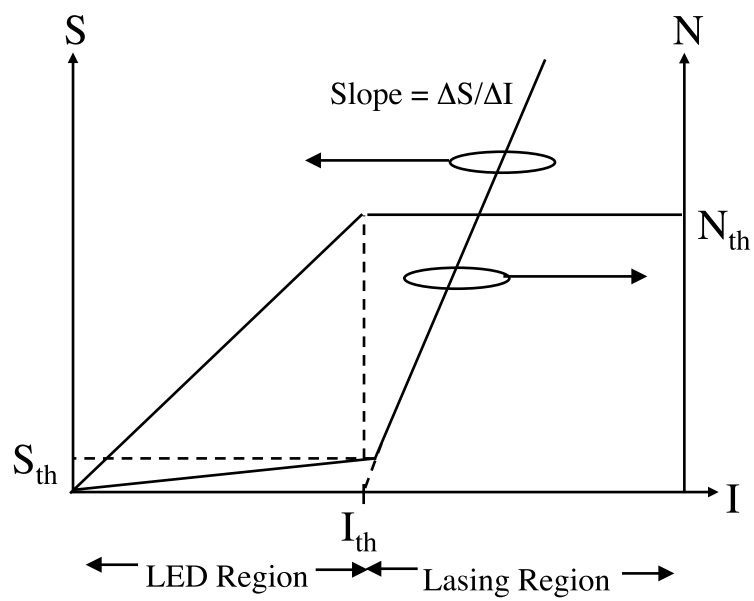
⇨ using the linear gain relation, and at threshold gain = total losses

$$N = N_{tr} + \frac{g_p(N)}{a}$$

$$N_{th} = N_{tr} + \frac{g_p(N_{th})}{a} = N_{tr} + \frac{\alpha_{total}}{a}$$

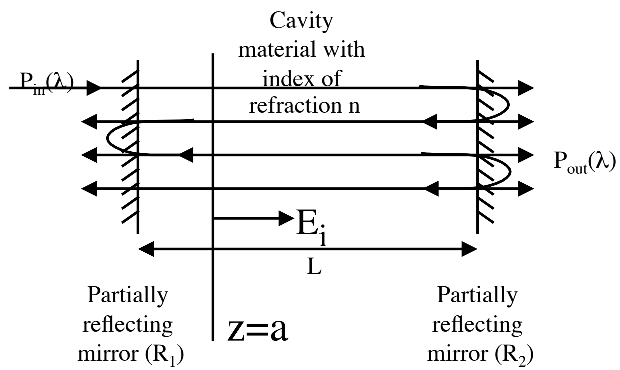
⇨ At and above threshold, with non-zero S and assuming linear operation (ε = 0)

$$S = \frac{I}{qVG(N_{th})} - \frac{R(N_{th})}{G(N_{th})} = \frac{I - qVR(N_{th})}{qV\alpha_{total}} = \frac{I - I_{th}}{qV\alpha_{total}}$$



Fabry-Perot Cavities

⇨ The equivalent of an electronic comb filter, but for optical frequencies, the Fabry-Perot (FP) cavity is used for feedback in lasers and as optical filters



⇨The propagation constant for a plane wave propagating in the cavity is

$$k'(\omega) = k_0 n'_{eff} = k_0\sqrt{n^2 + \chi' - i\chi''} - i\frac{\alpha}{2} \cong k + k\frac{\chi'(\omega)}{2n^2} - ik\frac{\chi''(\omega)}{2n^2} - i\frac{\alpha}{2}$$

⇨ The output wave P_{out} can be written as

⇨

$$\begin{aligned}
E_{out} &= E_1 + E_2 + E_3 + \cdots \\
E_1 &= t_2 e^{-ik'(l-a)} E_i \\
E_2 &= (r_1 r_2 e^{-i2k'l}) t_2 e^{-ik'(l-a)} E_i \\
E_3 &= (r_1 r_2 e^{-i2k'l})^2 t_2 e^{-ik'(l-a)} E_i \\
E_4 &= (r_1 r_2 e^{-i2k'l})^3 t_2 e^{-ik'(l-a)} E_i
\end{aligned}$$

Fabry Perot Cavities

- Summing up the fields

$$E_{out} = t_2 e^{-ik'(l-a)} E_i \left\{ 1 + (r_1 r_2 e^{-i2k'l}) + (r_1 r_2 e^{-i2k'l})^2 + (r_1 r_2 e^{-i2k'l})^3 + \cdots \right\}$$
$$= \frac{t_2 e^{-ik'(l-a)}}{1 - r_1 r_2 e^{-i2k'l}} E$$

- Rewriting k'

$$k' = k + \Delta k + i^{(\gamma - \alpha)}\!\big/_{2} = k + k\frac{\chi'(\omega)}{2n^2} + i^{(\gamma - \alpha)}\!\big/_{2}$$

- Model the gain as (we will look at this in more detail next class)

$$\gamma = -k\frac{\chi''(\omega)}{n^2} = (N_2 - N_1)\frac{\lambda^2}{8\pi n^2 t_{spont}} g(\nu)$$

Fabry Perot Cavities

- Locating the optical source at the input (z=0)

$$E_{out} = \frac{t_1 t_2 e^{-i(k+\Delta k)l} e^{(\gamma-\alpha)l/2}}{1 - r_1 r_2 e^{-i2(k+\Delta k)l} e^{(\gamma-\alpha)l}} \left(E_i\right)_{outside}$$

- When the denominator goes to zero, the cavity oscillates and we can write the amplitude and phase conditions for oscillation

$$r_1 r_2 e^{-i2(k+\Delta k)l} e^{(\gamma-\alpha)l} = 1$$

$$r_1 r_2 e^{(\gamma(\omega)-\alpha)l} = 1$$

$$2(k + \Delta k(\omega))l = 2m\pi, \ \text{m=1,2,3…}$$

- The amplitude condition can be written as

$$\gamma_t(\omega) = \alpha - \frac{1}{l}\ln r_1 r_2$$

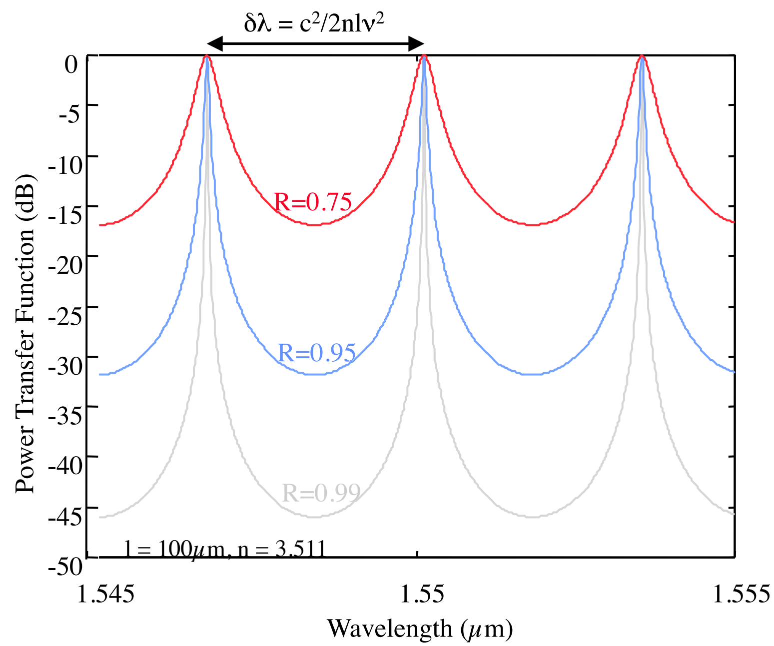
Fabry-Perot Cavities

- For end mirror loss $\alpha_m = \frac{1}{2L}\ln\frac{1}{R_1R_2}$
- The oscillation condition becomes $2j\beta L + \alpha_m L = 2j\pi N$, where N is an integer denoting longitudnal mode number
- Defining cavity roundtrip gain $g_c = g_{net} - \alpha_m$
- The real part of oscillation condition gives $g_c = 0$
- And $\Gamma g_a - \alpha_i - \alpha_m = 0$
- Defining the total cavity loss as $\alpha_{total} = \alpha_i + \alpha_m$
- Then $g_c = g_{eff} - \alpha_{total} = 0$
- And the resonance condition for the cavity is

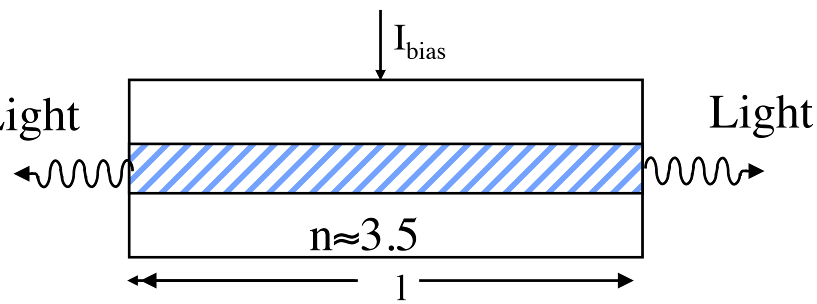
$$\lambda_n = \frac{2n'_{eff}(\lambda_n)L}{N}$$

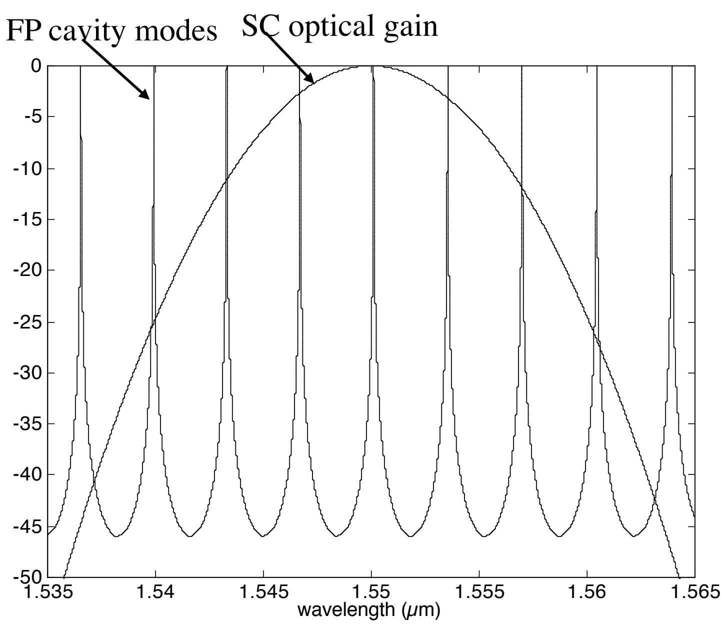
$$\Delta\lambda_m = \lambda_n - \lambda_{n+1} \approx \frac{\lambda_N^2}{2n_{g,eff}L}$$

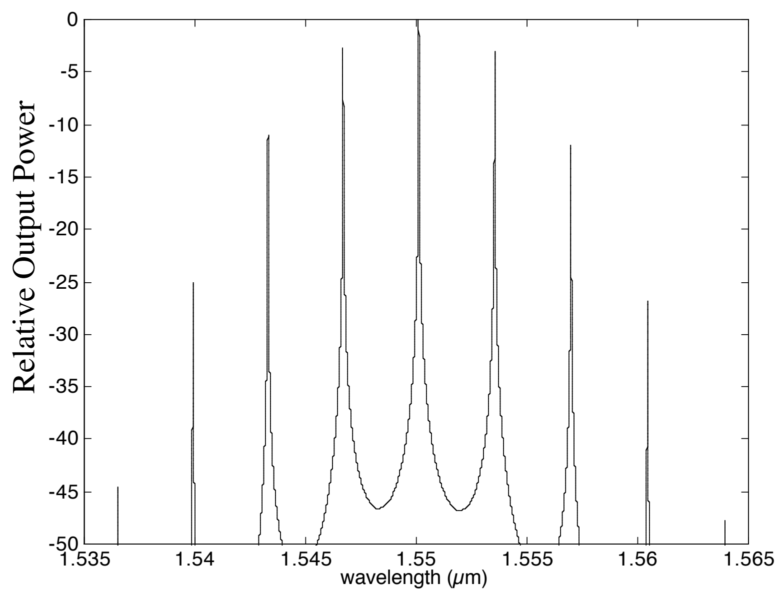
$n_{g,eff}$ = effective group index



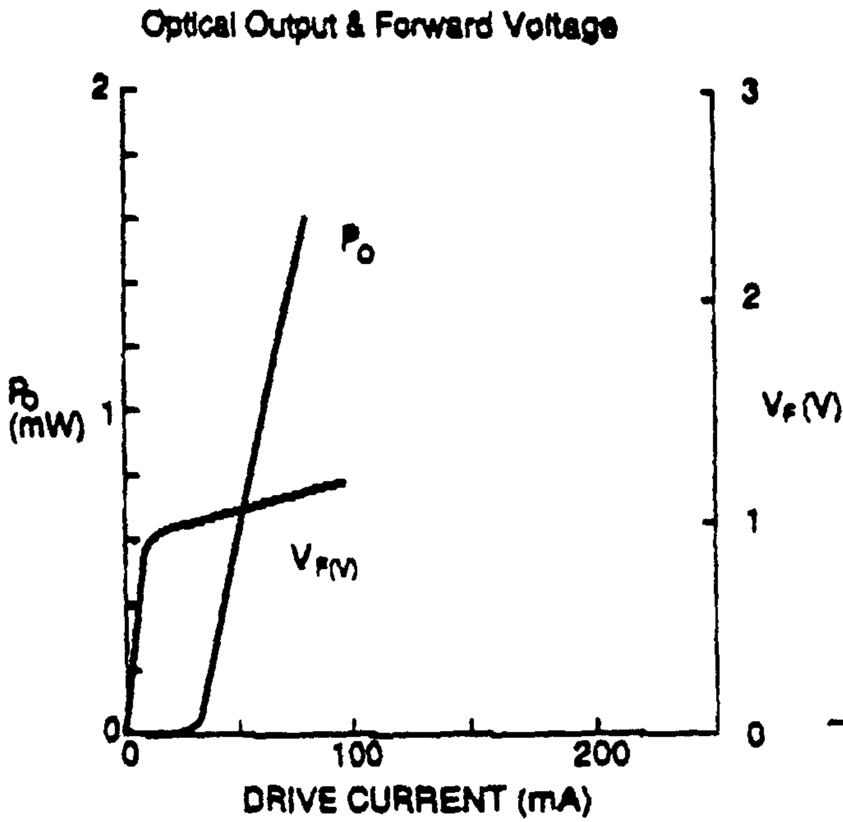
Multimode Fabry-Perot SC Lasers

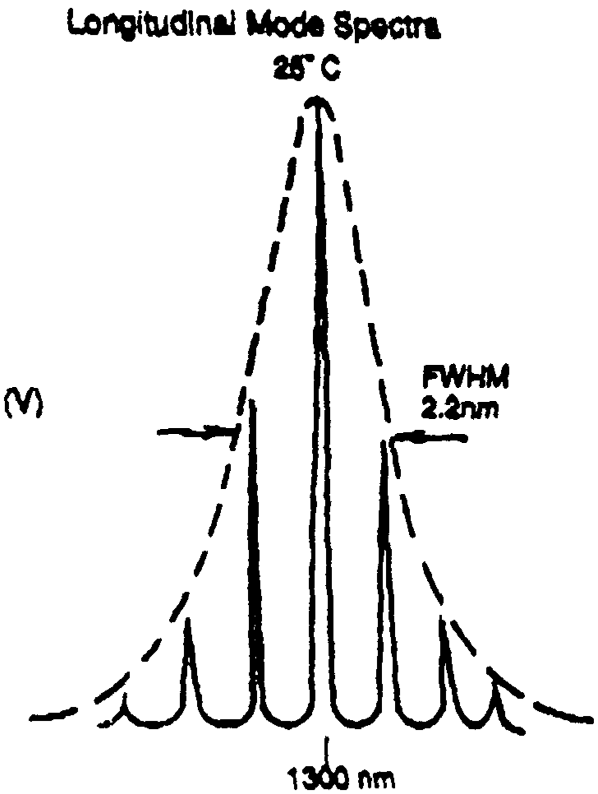






FP Laser Output Characteristics





•1.3 μm multimode lasers are good for bit rates < 2Gbs and distances up to 100 km.